

Optical transfer function shaping and depth of focus by using a phase only filter

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The design of a desired optical transfer function (OTF) is a common problem that has many possible applications. A well-known application for OTF design is beam shaping for incoherent illumination. However, other applications such as optical signal processing can also be addressed with this system. We design and realize an optimal phase only filter that, when attached to the imaging lens, enables an optimization (based on the minimal mean square error criterion) to a desired OTF. By combining several OTF design goal requirements, each represents a different plane along the beam propagation direction, an imaging system with an increased depth of focus is obtained. Because a phase only filter is used, high energetic efficiency is achieved. © 2003 Optical Society of America

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1. Introduction

In modern optics diffractive elements play a major role. Diffractive optical element(s) (DOE)(s) can be designed to utilize functions that would be difficult or impossible to achieve by conventional optical elements. Moreover, DOEs are characterized by lighter weight, smaller dimensions, and lower costs as compared with their refractive or reflective counterparts.^{1–4} Unfortunately, DOEs are based on diffraction, and thus they are highly dispersive (i.e., wavelength sensitive). For this reason DOEs are usually used in systems based on monochromatic illumination. Alternatively, one can use the highly depressive nature of DOEs to perform separations of wavelengths.

In this paper we present what we believe is a novel approach for enlarging the depth of focus (DOF) of an imaging system by use of a special DOE. A DOF defines the maximal acceptable deviation from the focal plane of an imaging system based on a resolution criterion. DOF is inversely proportional to the aperture size—large aperture results in a smaller DOF. There are several approaches that attempt to enlarge the DOF of imaging systems. One interesting approach is related to encoding the aperture plane with a cubic phase element.^{5–8} This approach

produces an image that after proper digital postprocessing produces an in-focus image.

The proposed technique is based on an iterative design of a phase only filter that is attached to the imaging lens. The iterative algorithm suggested in this paper is based on the Gerchberg–Saxton (GS) algorithm.^{9–11} The desired range for which the focus should be maintained is divided into N planes. The optical transfer function (OTF) of each plane is now calculated, assuming that the aperture size is half of the actual aperture size to be used. This way, an improved DOF is obtained. Our goal is to design an OTF that enables the achieving of exactly such a DOF while keeping the original aperture dimensions, i.e., allowing much more light to be transferred by the imaging system. Note that the factor of half was chosen arbitrarily to demonstrate the increased DOF (the DOF of half the aperture is approximately 4 times larger). Choosing a different factor could demonstrate an even larger DOF. In each iteration a phase only filter that generates the desired OTF of each transverse plane is computed. Then, N phase only filters that correspond to the N transverse planes are averaged and the result is converted into a phase only filter by setting its amplitude to be one, i.e., by neglecting the amplitude contribution. The obtained result is used as a starting point for the next iteration, and the process continues until the variation of the obtained results is smaller than the limit defined as convergence. The suggested algorithm allows setting a nonuniform weighting to the contributions from each plane to control the trade off between resolution and DOF.

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An analytic approach for the design of a desired OTF was already suggested.¹² However, the iterative approach provides much more design freedom, and allows obtaining improved results. Note that iterative approaches are commonly used and previously discussed for filtering applications.¹³

Section 2 describes the effect of out-of-focus imaging on the OTF. Section 3 discusses theoretical aspects of the suggested approach. Computer simulations and experimental results are presented in Sections 4 and 5, respectively. Section 6 concludes the paper.

2. Optical Transfer for Out-of-Focus Function Imaging

The influence of out-of-focus imaging on the OTF is well known and can be found in many textbooks.¹ Nevertheless, we feel that a short description is still helpful.

When the imaging system is diffraction limited, the amplitude point-spread function consists of the Fraunhofer diffraction pattern of the exit pupil, centered on the ideal image plane. However, when the observation plane is out of focus, wave-front errors exist. This case can be described by an exit pupil, which is illuminated by a perfect spherical wave. By tracing a ray backwards from the ideal image point to the coordinates (x, y) in the exit pupil, the aberration function $W(x, y)$ is the path-length error accumulated by that

plane to the perfect imaging plane. The path-length error $W(x, y)$ can then be determined by subtracting the ideal phase distribution from the actual phase distribution

$$kW(x, y) = \frac{\pi}{\lambda Z_a} (x^2 + y^2) - \frac{\pi}{\lambda Z_i} (x^2 + y^2). \quad (2)$$

The path-length error is thus given by

$$W(x, y) = \frac{1}{2} \left(\frac{1}{Z_a} - \frac{1}{Z_i} \right) (x^2 + y^2). \quad (3)$$

By assuming a square aperture of width $2w$, the maximal path-length error at the edge of the aperture along the x or y axes is given by

$$W_m = \frac{1}{2} \left(\frac{1}{Z_a} - \frac{1}{Z_i} \right) w^2. \quad (4)$$

The number W_m is a convenient indication of the severity of the focusing error. By use of the definition of W_m , the path-length error can be expressed by

$$W(x, y) = W_m \frac{x^2 + y^2}{w^2}. \quad (5)$$

We can now estimate the effect of the focusing aberration on the OTF (applicable for incoherent imaging). An ideal, aberration free OTF is given by

$$\text{OTF}(f_x, f_y) = \frac{\iint P\left(x + \frac{\lambda Z_i f_x}{2}, y + \frac{\lambda Z_i f_y}{2}\right) P\left(x - \frac{\lambda Z_i f_x}{2}, y - \frac{\lambda Z_i f_y}{2}\right) dx dy}{\iint P(x, y) dx dy}, \quad (6)$$

ray as it passes from the reference sphere to the actual wave front. The error can be positive or negative, depending on whether the actual wave front lies to the left-hand side or to the right-hand side (respectively) of the reference sphere. Thus, the phase distribution across the exit pupil is of the form of

$$\phi(x, y) = \frac{\pi}{\lambda Z_a} (x^2 + y^2), \quad (1)$$

where Z_a is the distance between the aperture plane to the observation plane. Generally speaking, $Z_a \neq Z_i$, where Z_i is the distance between the aperture

where $P(x, y)$ is the pupil function and (f_x, f_y) are the coordinates of the OTF.

Owing to the aberration, the effective pupil function is now multiplied by the phase delay caused by the aberration:

$$P_g(x, y) = P(x, y) \exp[jk W(x, y)]. \quad (7)$$

Now, it is possible to find the OTF of a system in the presence of the aberration, by substitution of Eq. (7) into Eq. (6):

$$\text{OTF}(f_x, f_y) = \frac{\iint_{A(f_x, f_y)} \exp\left\{jk \left[W\left(x + \frac{\lambda Z_i f_x}{2}, y + \frac{\lambda Z_i f_y}{2}\right) - W\left(x - \frac{\lambda Z_i f_x}{2}, y - \frac{\lambda Z_i f_y}{2}\right) \right]\right\} dx dy}{\iint_{A(00)} dx dy}, \quad (8)$$

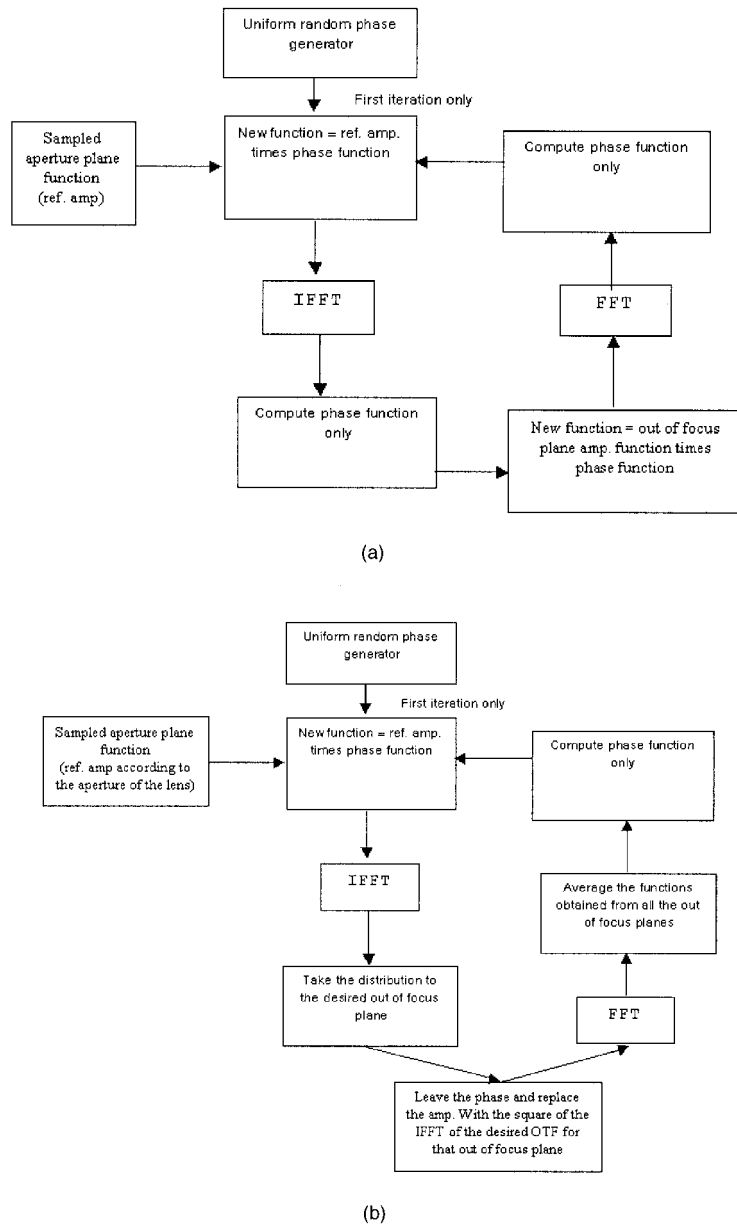


Fig. 1. (a) Schematic drawing of phase of the Gerchberg-Saxton algorithm, (b) the specific algorithm applied for the presented iterative approach.

where $A(f_x, f_y)$ is the area of integration, i.e., the overlapping area between two shifted apertures [the shift is a function of (f_x, f_y)]. $A(0, 0)$ is the area of the lens aperture (the shift is zero).

By substituting Eq. (5) into Eq. (8) and performing several straightforward manipulations one obtains

$$\begin{aligned} \text{OTF}(f_x, f_y) &= \Lambda\left(\frac{f_x}{2f_0}\right) \Lambda\left(\frac{f_y}{2f_0}\right) \\ &\times \text{sinc}\left[\frac{8W_m}{\lambda} \left(\frac{f_x}{2f_0}\right) \left(1 - \frac{|f_x|}{2f_0}\right)\right] \\ &\times \text{sinc}\left[\frac{8W_m}{\lambda} \left(\frac{f_y}{2f_0}\right) \left(1 - \frac{|f_y|}{2f_0}\right)\right], \quad (9) \end{aligned}$$

where Λ denotes a triangular function.

It can be easily seen that a diffraction-limited OTF is indeed obtained for the case of $W_m = 0$. However, for values of $W_m \geq \lambda/2$, which represent a significant defocusing error, sign reversal of the OTF occurs. As can be seen, a gradual attenuation of contrast and a number of contrast reversals are obtained for high spatial frequencies.

3. Description of the Algorithm and Filter Design

This GS algorithm is used for phase retrieval of a wave function whose intensity in the lens and the imaging planes is known. The basic algorithm is an iterative procedure that is shown schematically on Fig. 1(a).

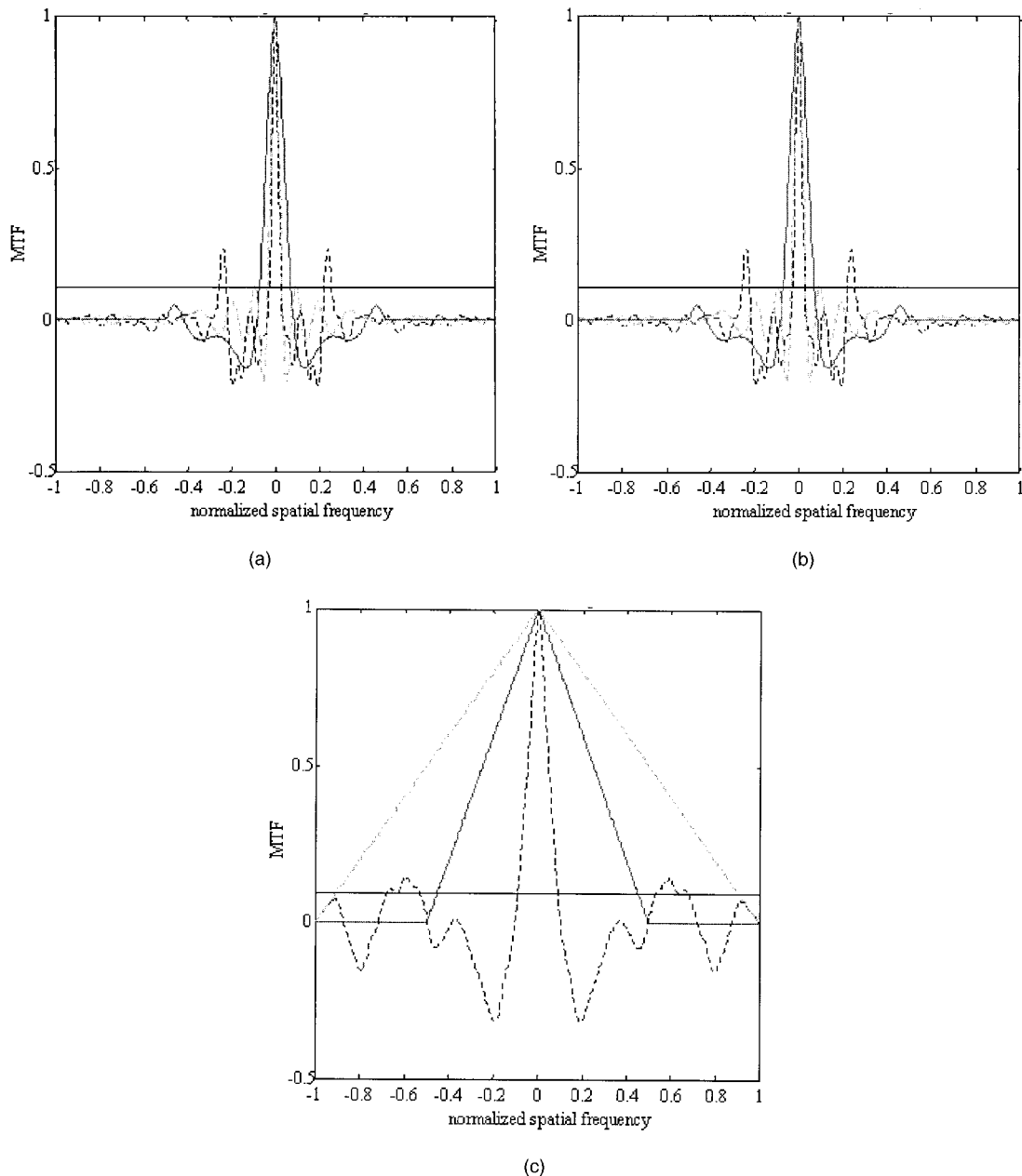


Fig. 2. (a) Three OTF for comparison at the left-hand side edge of the DOF range, (b) three OTF for comparison at the right-hand side edge of the DOF range, (c) three OTF for comparison at the focal plane.

The procedure starts by the choosing of a random phase function that is multiplied by a rectangular amplitude function representing the shape of the lens aperture. Inverse fast Fourier transform (IFFT) of this synthesized complex discrete function is then performed. The phase of the obtained result is kept while the amplitude is set to be the square of the inverse Fourier transform of the desired OTF distribution that corresponds to twice the smaller aperture. The distributions (computed for each of the N out-of-focus planes) are Fourier transformed. The result is averaged, and the amplitude is set to be a rectangular function as in the previous iteration. A weighted average can be obtained by multiplying the

contribution of each plane by a different weighting coefficient [see Fig. 1(b)]. The process is repeated until the variations from one iteration to the other are bounded. By varying the weights of the different planes and by changing the desired OTF distribution constrained per each plane, one may determine the resolution of the designed system obtained within different positions in the DOF range.

During the iterative algorithm, Eq. (9) is used to generate the real OTF as well as the desired OTF. The desired OTF has an aperture width of half the width of the real aperture of the imaging lens (smaller apertures result in a larger depth of focus range). By applying such a constraint one obtains an in-

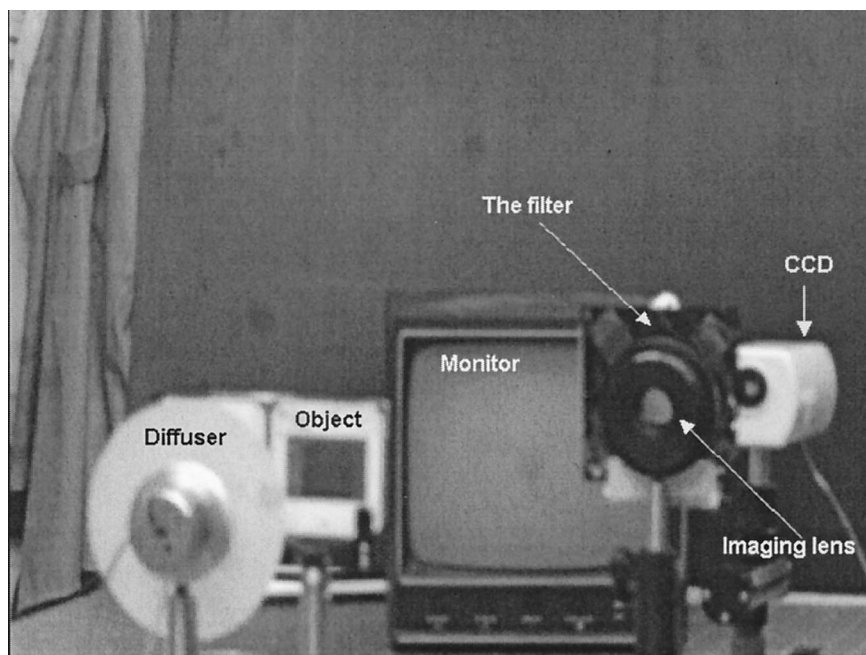


Fig. 3. The experimental setup.

creased depth of focus (that corresponds to half the width aperture), while maintaining high energetic efficiency (corresponding to a full width aperture) because a phase-only filter is used.

4. Computer Simulations

As a test case, we picked up 11 uniformly spaced out-of-focus planes. The focal length used for the simulation and the phase-mask design was 20 cm. The diameter of the lens was 2 cm. The DOF range was ± 2 mm around the in-focus plane. The wavelength was 630 nm.

Simulation results are given in Fig. 2. As can be seen, three OTFs are plotted, one on top of the other. The three curves of Fig. 2(a) correspond to the desired OTF (with aperture of half the diameter), the OTF obtained when the filter is not used, and the OTF obtained by using the suggested filter. The plot of the results is obtained at the left-hand side edge of the desired extended DOF range. As a criterion for resolving this information we chose a contrast threshold of 0.1. Thus a line representing this value was added to the figures. By observing the results one may notice that the real OTF cutoff frequency is smaller without the filter than with the use of the filter. This result is expected because resolution and DOF are inversely proportional. Indeed, the DOF of the system with the filter is even larger than twice the DOF of the system with a twice smaller aperture.

In the other edge of the extended DOF range one may see that no improvement is achieved. In Fig. 2(b) it can be easily seen that the dashed curve (OTF with the use of the filter) and the dashed-dotted curve (OTF without the filter) are nearly the same, and their spatial cutoff frequencies are identical. Figure 2(c) shows all three OTF at the focus plane. As ex-

pected, the real OTF is twice as wide as the desired OTF because its aperture is twice as wide. Note that those two OTFs are ideal triangles, as expected. However, the OTF that is received by use of the filter is much thinner and distorted. This is expected as well, because the reference in the algorithm was the desired OTF, and that is thinner than the physical aperture of the real lens. In addition the distortions are also caused by the restrictions to obtain the increased DOF.

5. Experimental Results

To further verify the validity of the proposed approach, an optical experiment was carried out. The parameters used for the experiment are similar to the one specified in the simulation in Section 4. A phase filter with 8 gray phase levels was generated by using a lithographic recording. The 8-level DOE was obtained with 3 binary masks. The masks were created by use of a Dolev plotter with 3600 dpi and then reduced by a factor of 10 to a milimask by use of a high-resolution imaging setup. Moving the imaged object generated the introduced defocusing. The movement range was of a few centimeters. The edge of the range at which a focused image was obtained was allocated. Fig. 3 depicts the setup of the experiment. The setup contained a light source, an imaging lens with the phase filter attached to it, and a CCD camera. The image of the object is obtained by the CCD and displayed on the monitor. The object is a transparent film with regions of different spatial frequencies. Because a He-Ne light source, which generates monochromatic illumination, is a coherent light source, a diffuser was added to create incoherent illumination. The results are shown in Figs. 4(a)–4(c).

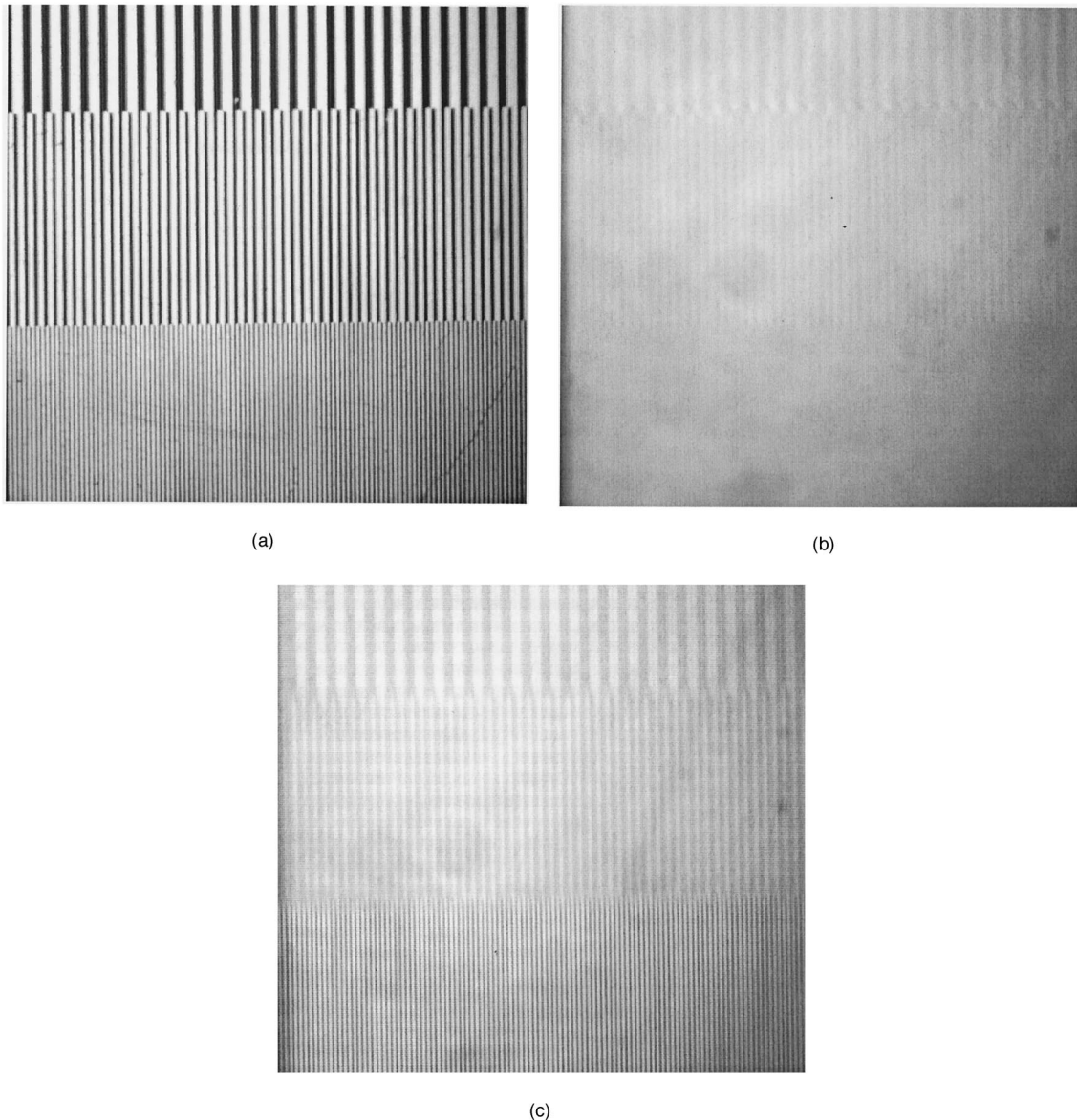


Fig. 4. (a) Perfectly focused image (without the filter), (b) misfocused image without the filter, (c) with the filter.

Figure 4(a) presents a perfect imaging that was obtained by focusing the CCD on the perfect imaging plane ($Z = Z_i$). In this case, no filter was used. Fig. 4(b) was taken by focusing the CCD on an out-of-focus plane. Again, no filter was used. As can be seen, it is very difficult to identify the high frequencies. Figure 4(c) is identical to Fig. 4(b). The only difference is the use of the suggested filter. One can clearly see that using the filter improves the image quality. The high frequency, that cannot be observed without the filter is clearly resolved after attaching the filter to the imaging lens. The two stripes with the lower spatial frequencies that are hardly seen without the filter due to low contrast can be clearly identified, because better contrast is now achieved. The experiment demonstrates that employing the filter increases the depth-of-focus range. In summary, the experimental results have demonstrated a DOF im-

provement of approximately one order of magnitude (The DOF range was more than 3.3 cm).

Let us now add several insightful remarks regarding the suggested technique. The number and the concentration of the planes used for the computation of the phase-only filter is determined such that the DOF obtained for a single plane will be larger than the separation distance between adjacent planes. Obviously, having too many constraint planes increases the number of constraints, and eventually the averaging of the various OTFs cause the degradation of the anticipated performance obtained due to a constraint of a single plane. Because no digital processing, such as inverse filtering, is applied, the sensitivity to noise is not significant. Spectrally, wideband illumination will smear the image, however the smearing effect is similar to the sensitivity of a regular DOE dealing with a single plane.

6. Conclusions

In this paper we have demonstrated what is to our knowledge a novel technique for an iterative OTF design to obtain an imaging system with an improved depth of focus. Computer simulations as well as experimental results have demonstrated the improved depth of focus obtained by use of the suggested approach.

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