## Simultaneous multicolor image formation with a single diffractive optical element

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A design for a novel diffractive optical element (DOE) that can reconstruct three different intensity patterns when it is illuminated by three different wavelengths is presented. If the chosen wavelengths are red, green, and blue, full-color reconstruction capability is obtained. Reconstruction is achieved in the near field (Fresnel domain). Computer simulation results as well as experimental evidence are presented, proving the capabilities of this novel DOE design procedure. © 2001 Optical Society of America OCIS codes: 050.1970, 260.2030, 330.1690.

Diffractive optical elements (DOEs) play a major role in various applications, such as beam shaping,¹ optical data processing,² and optical interconnections.³ These elements are based mainly on a surface-relief pattern that affects the phase modulation, thus controlling the waveform behavior. A prominent feature of these elements is high wavelength sensitivity. In contrast with conventional optical elements, in which wavelength sensitivity is caused only by the dependence of the refractive index in the wavelength, DOEs are inherently wavelength dependent, since the phase modulation via surface-relief variations is wavelength dependent (the wave number is inversely proportional to the wavelength) and the diffraction angles increase with the wavelength (approximately linearly).

The wavelength sensitivity of DOEs is usually considered to be undesired and harmful. It prevents one from utilizing DOEs in many applications in which white-light illumination is involved, since the desired reconstruction will be severely degraded for wavelengths that are different than the designed single wavelength. However, for applications in which different wavelengths should be distinguished, a DOE is the natural choice. DOEs are used for spectral separation, tunable laser systems, measuring surface profiles and many more applications that rely on wavelength separation.

A well-known color-separation DOE was designed by Dammann. He designed a steep grating with a modulation depth much larger than  $2\pi$  and succeeded in obtaining far-field reconstructions of different colors in the -1, 0, and +1 diffraction orders. It should be mentioned, however, that the filter (grating) that was designed according to this approach does not contain any information other than on color separation, and thus only one spot is reconstructed at each diffraction order. Only the combination of such a filter with a color illuminated object will yield true color separation. Moreover, the requirement for the fabrication of such a steep grating is difficult to fulfill.

It has already been shown that a kinoform can reconstruct two differently designed spot patterns when it is illuminated by green and red light. Impressive reconstruction was achieved in the near field as well as in the far field. To obtain far-field reconstruction it was

necessary to increase the phase modulation so that it represents several  $2\pi$  cycles. Alternatively,  $2\pi$  modulation can still be used; however, the efficiency is lower (high dc peak). The near-field reconstruction was easier to achieve, and phase modulation of more than  $2\pi$  was unnecessary, since near-field reconstruction is based on both wavelength-separation mechanisms, as mentioned above. The reason for this is the translation of different diffraction angles into different focusing powers (as in the case of a diffractive lens), whereas, in the far field, different diffraction angles influence the scale only. The kinoform described above was designed by use of the optimal rotation angle method,9 which optimizes each and every pixel, thus achieving sufficient results but needing very great computational effort.

This Letter describes an attempt to go a step further: to design a DOE that reconstructs three (rather than two) different objects when it is illuminated by red, green, and blue light.  $2\pi$  modulation was used, and thus the DOE could be easily fabricated. Reconstruction was achieved in the near field. The importance of three-color reconstruction is unquestionable: Since blue, green, and red are the three basic colors, a reconstruction of full-color images consisting of the three different images can be achieved. The filter was designed with a modified version of the iterative ping-pong algorithm,  $^{10,11}$  and thus computation time was very short, less than 1 s per iteration, on a standard PC.

The phase modulation of the *m* pixel is given by

$$\phi_m^i = \frac{2\pi}{\lambda_i} (n-1) h_m, \qquad (1)$$

where  $\lambda$  is the wavelength, i is the wavelength index (for our three-color illumination case i is 1, 2, and 3 for blue, green, and red light, respectively), n is the substrate refractive index (assumed to be unaffected by the wavelength), and  $h_m$  is the actual depth of the m pixel. The phase-modulation difference between  $\lambda_i$  and  $\lambda_j$  is thus given by

$$\Delta \phi_m^{i,j} = 2\pi (n-1) h_m \left( \frac{1}{\lambda_i} - \frac{1}{\lambda_j} \right)$$
 (2)

For our case, we assume that  $\lambda_1=0.442~\mu\mathrm{m}$  (blue),  $\lambda_2=0.532~\mu\mathrm{m}$  (green), and  $\lambda_3=0.633~\mu\mathrm{m}$  (red). We design the DOE so that the maximal value of  $2\pi$  phase modulation can be achieved when one is using green light; thus  $\phi_{\mathrm{max}}^3=0.84(2\pi)$ ,  $\phi_{\mathrm{max}}^2(2\pi)$ ,  $\phi_{\mathrm{max}}^1=1.2(2\pi)$ , and  $\Delta\phi_{\mathrm{max}}^{1.2}=0.2(2\pi)$ ,  $\Delta\phi_{\mathrm{max}}^{1.3}=0.36(2\pi)$ ,  $\Delta\phi_{\mathrm{max}}^{2.3}=0.16(2\pi)$ . If we keep in mind that, on average, much lower phase-difference values will be obtained, the difficulty in achieving color separation based only on this mechanism becomes evident.

Starting with a phase-only filter H(x, y), we now perform free-space propagation from the Filter plane to a distance  $Z_0$  from it. By virtue of the Fresnel approximation, we obtain

$$|f_{i}(x', y')|^{2} = \left| \frac{1}{\lambda_{1} Z_{0}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H(x, y) \right|$$

$$\times \exp \left[ i \frac{\pi}{\lambda_{i} Z_{0}} (x^{2} + y^{2}) \right]$$

$$\times \exp \left[ -i \frac{2\pi}{\lambda_{i} Z_{0}} (xx' + yy') \right] dx dy \right|^{2}. \quad (3)$$

It can be seen that, besides the different phase modulation that results, changing the wavelength is the same as changing the propagation distance. Since it is already known that one filter can generate two different images in two different transversal planes,12 it should be possible to reconstruct two different images in the same plane when the DOE is illuminated by two different wavelengths. Going one step further, since two wavelengths are not sufficient for achieving red-green-blue reconstruction, we focus our efforts toward achieving reconstruction of three different images in the same plane when the DOE is illuminated by three different wavelengths (for example, red, green, and blue). This reconstruction is to be combined with an efficient and fast algorithm that should yield good results after a very short time.

Can the three intensity distributions  $f_1(x', y')$ ,  $f_2(x', y')$ , and  $f_3(x', y')$  represent three desired, non-identical intensity functions in a common plane? We release all restrictions on the displayed phase but demand that the amplitude of those distributions should match the desired output functions to be reconstructed.

The phase-only filter, H(x,y), that can generate the desired distributions can be found by use of an iterative algorithm if we bear in mind that the amplitude distributions of  $f_i$  (i=1,2,3) are the constraints and that their phase distributions are free parameters with no restrictions. The proposed iterative procedure belongs to the family of the well-known ping-pong algorithm. Adapting this algorithm to our needs, we find that the three constraints in the reconstruction plane must all be satisfied by the same function H(x,y). In Fig. 1, a flowchart of the iterative algorithm is presented.  $W_l$  stands for the weighting coefficient of the averaging.

Different criteria can be used to examine the quality of the reconstructed images. We used the minimum absolute intensity error (MIE) criterion. Since the energy of both the desired and the obtained reconstructions should be equal to the energy of the phase-only filter (Parseval's theorem), the MIE is defined as

$$MIE^{i} = \sum_{m} \sum_{n} ||A_{m,n}^{i-\text{des}}|^{2} - |A_{m,n}^{i-\text{obt}}|^{2}|, \qquad (4)$$

where the total energy is normalized to unity:

$$\sum_{m} \sum_{n} |A_{m,n}^{i-\text{des}}|^2 = \sum_{m} \sum_{n} |A_{m,n}^{i-\text{obt}}|^2 = 1.$$
 (5)

We define total MIE as the average of all three-color  $MIE^{i}$ :

$$MIE = \frac{1}{3} \sum_{i=1}^{3} MIE^{i}.$$
 (6)

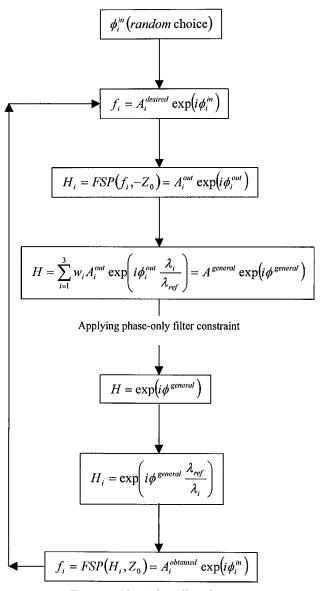


Fig. 1. Algorithm flowchart.

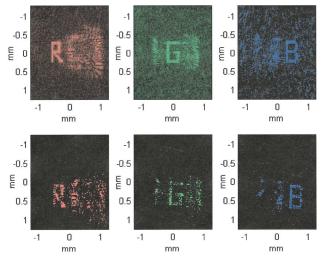


Fig. 2. Reconstruction of the three-color channels. Top row, simulation results; bottom row, experimental results.

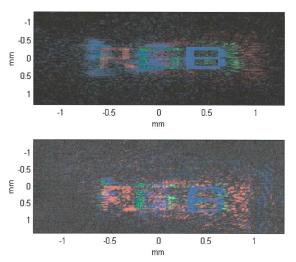


Fig. 3. Reconstruction for the case of three-color simultaneous illumination. Top row, simulation results; bottom row, experimental results.

We can stop the algorithm by limiting the number of iterations, achieving a reasonable MIE criterion, examining the reduction of the MIE, or any combination of these criteria. Other decision criteria can be applied as well.

As a test case, we chose the letters R, G, and B, to be reconstructed in red, green, and blue light, respectively, in the same plane and side by side. We assumed a phase-only filter consisting of  $128 \times 128$  pixels. The computer simulation assumes  $20 - \mu m$  rectangular pixels, and a propagation distance of 11 cm.  $\lambda_{\rm ref} = 0.532~\mu m$  is assumed, so maximal phase modulation of  $2\pi$  can be achieved for green light. Fifty iterations were carried out. Computation time with a Pentium III 450-MHz processor was less than 1 m. A total MIE of 0.0355 was achieved. For comparison, a MIE of 0.007 was achieved for a single-color (green) case. Convergence was very fast, and after 10 iterations the MIE was no more than 120% of its final value. The results were influenced

very little by the initial choice of the phase function as long as a random phase was used. The three computer-reconstructed images can be seen in the top section of Fig. 2. A binary phase element has been assumed. The summation of the three intensity patterns, which simulates a case in which all three lasers illuminate the DOE simultaneously, is shown in the top part of Fig. 3 (a square root was used for visualization purposes). A continuous phase profile has been assumed. As can be seen, for such a case red-green-blue reconstruction can be achieved.

To verify further the capabilities of the proposed approach, we carried out an optical experiment. The dimensions of the DOE were similar to those used for the computer simulations. Only one binary mask was used, resulting in a binary phase-only filter. The reconstruction achieved by illumination of the element with red, green, and blue light is shown in the bottom row of Fig. 2. The case of three-color simultaneous illumination is shown in the bottom part of Fig. 3. As can be seen, there is a significant reconstruction error that is due to the significant quantization noise. These experimental results can be improved by use of a multilevel DOE.

In conclusion, a novel diffractive optical element (DOE) that is able to reconstruct different intensity patterns when it is illuminated by several different wavelengths is presented. In particular, the case of red-green-blue illumination is considered. The element is implemented as a phase-only filter, and thus high diffraction efficiency can be achieved. The chosen wavelengths are red, green, and blue. Since any color can be created by proper combination of these colors, full-color reconstruction can be achieved. Reconstruction is achieved in the near field (Fresnel domain). The phase modulation was only  $2\pi$  (for green light), whereas other approaches require higher phase modulation. Computer simulation results as well as an optical experiment were carried out, proving the capabilities of this novel DOE.

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