Tight focusing of spatially variant vector optical fields with elliptical symmetry of linear polarization

Gilad M. Lerman[1,2] and Uriel Levy[1,3]

[1]Department of Applied Physics, The Benin School of Engineering and Computer Science, The Hebrew University of Jerusalem, Jerusalem, 91904, Israel

[2]Corresponding author: lermag@pob.huji.ac.il

[3]ulevy@cc.huji.ac.il

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We study the tight-focusing properties of spatially variant vector optical fields with elliptical symmetry of linear polarization. We found the eccentricity of the incident polarized light to be an important parameter providing an additional degree of freedom assisting in controlling the field properties at the focus and allowing matching of the field distribution at the focus to the specific application. Applications of these space-variant polarized beams vary from lithography and optical storage to particle beam trapping and material processing.

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The challenging quest for achieving the most sharply focused light spot has recently made significant progress by the use of cylindrical-vector beams. In particular, it was shown [1] that a radially polarized light beam can be focused into a tighter spot compared with a linearly polarized beam. These beams can be used in applications such as lithography, optical data storage, microscopy, material processing, and optical trapping. The properties of the optical field near the focus of a high numerical aperture (NA) lens were investigated in numerous recent papers [2–4]. The symmetry of the polarization of the radially polarized field and its orthogonal state, the azimuthally polarized field, leads to unique properties of the field at the focal plane, e.g., strong longitudinal component and sharper focal spot for a radially polarized field and donut beam for an azimuthally polarized field.

Previous works investigated the focusing properties of radially and azimuthally polarized light beams that hold a cylindrical symmetry, or cylindrical vector beams that can be decomposed into a linear, space-invariant superposition of radial and azimuthal polarization components. In this Letter, we investigate generalized vector beams that do not hold a cylindrical symmetry. Specifically, we calculate the optical field distribution at the focus of a high NA lens illuminated with a linearly polarized light beam having an elliptical symmetry of polarization, i.e., the polarization vector of the incident field is directed so as to form a set of concentric ellipses with constant eccentricity in space. The polarization vector can be chosen to follow a direction perpendicular or parallel to the ellipses (generalization of radially and azimuthally polarized light, respectively). We define these two polarization states as linearly polarized with elliptical radial symmetry (LIPERS) and linearly polarized with elliptical azimuthal symmetry (LIPEAS). From the ellipse equation we find the direction of the LIPEAS field to be $\tan\theta = \mp bx/a^2\sqrt{1-(x^2/a^2)}$, where a is the semi-major axis and b is the semi-minor axis of the ellipse and x is the lateral horizontal coordinate. The LIPERS field is directed perpendicular to θ. The eccentricity of an ellipse given by $\epsilon = \sqrt{1-(b^2/a^2)}$ can be varied from 0 (equivalent to radial or azimuthal polarization) to 1, thus providing an additional degree of freedom to control the optical field properties at the focus. Figure 1 shows a schematic diagram of a LIPEAS field with $\epsilon = 0.8$.

The basic approach for analyzing the optical field near the focus of a high NA lens was introduced by Richards and Wolf [5], where they considered a case of a uniformly polarized light beam, and was later adopted for the case of radially and azimuthally polarized light beams [3]. Our LIPERS or LIPEAS optical fields can be decomposed by using either the cylindrical or Cartesian coordinate system, and the obtained components can then be used to calculate the vector field at the focus based on the expressions given in [3,5], respectively, taking into account a major difference as detailed below.

Figure 2 shows the geometry of the problem. The electric field of the illumination beam can be expressed as

$$\overrightarrow{E^{(i)}} = E_x^{(i)}\overrightarrow{e_x} + E_y^{(i)}\overrightarrow{e_y}, \tag{1}$$

where $\overrightarrow{e_x}$ and $\overrightarrow{e_y}$ are unit vectors along the x and the y directions, respectively, and $E_x^{(i)}, E_y^{(i)}$ are their relative amplitudes, which in general are functions of θ and ϕ. Assuming circular aperture, and using [5], the field components at a point (ρ_f, ϕ_f, z_f) near the focus resulting from the initial x-polarized field are given by

$$\left.\begin{aligned}E_{xx}^{(f)}&=\frac{-iA}{\pi}\int_0^{\theta_m}\int_0^{2\pi}E_x^{(i)}(\theta,\phi)l_0(\theta)\cos^{1/2}\theta\sin\theta[\cos\theta+(1-\cos\theta)\sin^2\phi]\\&\quad\exp[ik(z_f\cos\theta+\rho_f\sin\theta\cos(\phi-\phi_f))]\mathrm{d}\phi\mathrm{d}\theta\\E_{xy}^{(f)}&=\frac{iA}{\pi}\int_0^{\theta_m}\int_0^{2\pi}E_x^{(i)}(\theta,\phi)l_0(\theta)\cos^{1/2}\theta\sin\theta(1-\cos\theta)\cos\phi\sin\phi\\&\quad\exp[ik(z_f\cos\theta+\rho_f\sin\theta\cos(\phi-\phi_f))]\mathrm{d}\phi\mathrm{d}\theta\\E_{xz}^{(f)}&=\frac{iA}{\pi}\int_0^{\theta_m}\int_0^{2\pi}E_x^{(i)}(\theta,\phi)l_0(\theta)\cos^{1/2}\theta\sin^2\theta\cos\phi\exp[ik(z_f\cos\theta+\rho_f\sin\theta\cos(\phi-\phi_f))]\mathrm{d}\phi\mathrm{d}\theta\end{aligned}\right\}, \quad (2)$$

where θ_m is the maximal ray angle passing through the lens (determined by the NA); $l_0(\theta)$ is the pupil apodization function, which is assumed to be a function of θ; k is the wavenumber, and $A=kf/2$, where f is the focal length of the lens. Similarly, the field components resulting from the initial y-polarized field can be calculated, where $E_x^{(i)}$ is replaced with $E_y^{(i)}$ and the expressions for $E_{xx}^{(f)}$ and $E_{xy}^{(f)}$ in the x-polarization account for $E_{yy}^{(f)}$ and $-E_{yx}^{(f)}$ in the y polarization, respectively. Since $E_x^{(i)}$ and $E_y^{(i)}$ are functions of both θ and ϕ, one needs to solve the double integral in Eq. (2). This is in contrast to previous works [3,5] where the amplitudes of the fields were invariant with respect to ϕ so that integration over ϕ could be replaced by a Bessel function. Finally, each component of the optical field near the focus is found by using superposition, e.g., $E_x^{(f)}=E_{xx}^{(f)}+E_{yx}^{(f)}$. The overall field is given by

$$\overrightarrow{E^{(f)}}=E_x^{(f)}\vec{e_x}+E_y^{(f)}\vec{e_y}+E_z^{(f)}\vec{e_z}, \quad (3)$$

where $\vec{e_z}$ is a unit vector in the z direction. Similar results can be obtained by generalizing the method of [3], where now

$$\overrightarrow{E^{(f)}}=E_\rho^{(f)}\vec{e_\rho}+E_\phi^{(f)}\vec{e_\phi}+E_z^{(f)}\vec{e_z}. \quad (4)$$

We also used this approach to validate our results.

For our calculations we choose the apodization function of [4]:

$$l_0(\theta)=\begin{cases}1 & \text{if } \sin^{-1}(0.1)\le\theta\le\sin^{-1}(\mathrm{NA})\\0 & \text{otherwise}\end{cases}, \quad (5)$$

with NA=0.8. The results are normalized by A. All length units are normalized by λ.

First, we calculated the electric field at the focal plane resulting from an LIPERS illumination field where $\epsilon=0$. As expected, the results were identical to [4]. Next, we repeated the process for $\epsilon=0.99$. Figure 3 shows the total energy density $w\propto|\overrightarrow{E^{(f)}}|^2$ at the focal plane. The incident field in this case is approaching a state where the two halves of the aperture plane are linearly polarized in opposite directions. As shown, the energy density distribution comprises two distinct peaks. The peak's intensities relative to the intensity of the area between them can be controlled by modifying the eccentricity of the illumination field. This can be used for a variety of applications, e.g., trapping particles in the two peaks and controlling the potential barrier between them so that interaction between particles in the existence of an external optical field can be investigated [6,7].

The new degree of freedom given by the eccentricity of the illumination field can be used together with the NA of the lens to yield a desired intensity distribution at the focal plane. A representative example is shown in Fig. 4, where a flattop intensity distribution is optimized for the case of NA=0.9. We found that a LIPERS illumination field with $\epsilon=0.87$ is optimal for this application, based on figure of merit defined as the width of the spot at 95% of the maximum intensity divided by the width at 50%. For our flattop spot

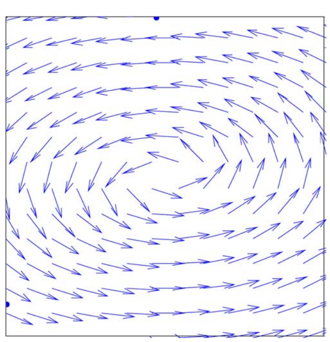

Fig. 1. (Color online) Schematic diagram showing the polarization direction of a beam with polarization vector creating a set of concentric ellipses in space with $\epsilon=0.8$.

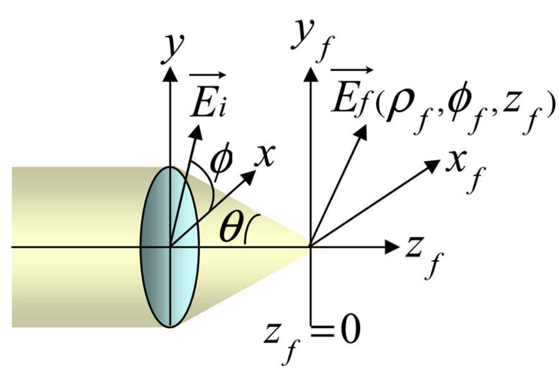

Fig. 2. (Color online) Schematic representation of the geometry of the problem.

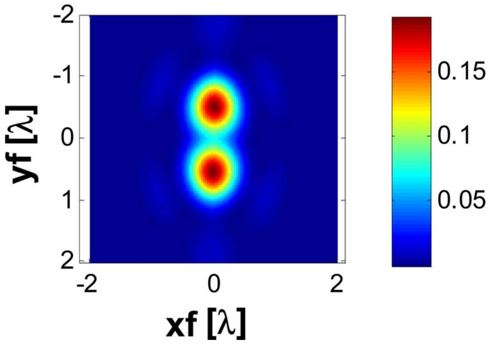

Fig. 3. (Color online) Total intensity distribution at the focus for a LIPERS illumination field ($\epsilon=0.99$, NA=0.8).

we obtained a value of 0.56 compared with 0.26 that is obtained for a Gaussian distribution.

Figure 5 shows the cross sections of the energy density distribution shown in Fig. 4, along the *xf* axis [Fig. 4(a)] and the *yf* axis [Fig. 4(b)]. As shown, we obtained a sharp line of equal energy density in one direction, with the length of about 2λ and the width of about λ. This pattern is of great importance for a variety of applications such as CD/DVD storage, where the pit length and the track pitch sizes are different. The flattop dimensions can be controlled by changing the NA and the eccentricity. Further decrease of the flattop width requires optimization of NA eccentricity, and apodization (see [1]).

The LIPERS and LIPEAS fields can be realized with space-variant inhomogeneous media on a subwavelength scale [8–10]. This concept was recently used for the formation of radially and azimuthally polarized beams [11,12]. We are currently working on providing experimental demonstration using this concept.

We studied the tight focusing properties of spatially variant vector optical fields with elliptical symmetry of linear polarization. The results show a strong dependence of the field near the focus on the eccentricity of the ellipses. We found the eccentricity to be an important parameter providing an additional degree of freedom assisting in controlling the field properties at the focus and allowing matching of the field distribution to the specific application. We consider the realization of such fields using the approach of subwavelength elements. It should be noted that the approach described in this Letter is general as it does not require the symmetry of the incident field, therefore allowing analyzing of the focal properties of arbitrary polarized fields. In particular, our approach is valid in cases where the direction of the incident polarized field is space variant in both the Cartesian and cylindrical coordinate systems. Applications of these space-variant polarized beams vary from lithography and optical storage to particle beam trapping and material processing.

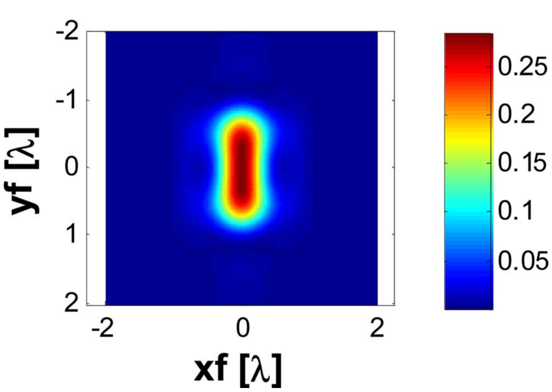

Fig. 4. (Color online) Flattop intensity distribution obtained for a LIPERS illumination field ($\epsilon=0.87$, NA=0.9).

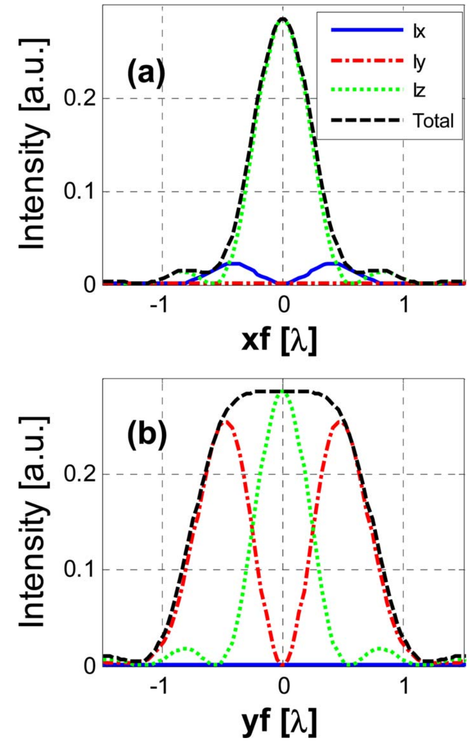

Fig. 5. (Color online) Cross sections of Fig. 4(a): along the *xf* axis and (b) along the *yf* axis.

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